

(6 pages)

Reg. No. :

Code No. : 20068 E Sub. Code : SMMA 62

B.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Sixth Semester

Mathematics — Core

NUMBER THEORY

(For those who joined in July 2017 onwards)

Time : Three hours

Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The value of $\binom{n}{0}$ is _____

- (a) 0 (b) n
(c) 1 (d) $n!$

2. Who say "Everything is number"? _____

- (a) Pythagoreans (b) Egyptians
(c) Greacean (d) Babylonians

3. gcd (8, 17) is _____

- (a) 2 (b) 8
(c) 1 (d) 0

4. If $\text{lcm}(a, b) = ab$, then gcd (a, b) is

- (a) 0 (b) 1
(c) ab (d) $(ab)^2$

5. The value of $\pi_{4,3}(89)$ is _____

- (a) 4 (b) 3
(c) 10 (d) 13

6. Which of the following is twin prime numbers?

- (a) 2, 3 (b) 5, 7
(c) 19, 23 (d) 79, 97

7. If $100x \equiv 0 \pmod{3}$, then the value of x is _____

- (a) 1 (b) 2
(c) 3 (d) 4

Page 2 Code No. : 20068 E

8. Which one of the following is correct? _____

- (a) $7 \equiv 0 \pmod{5}$ (b) $7 \equiv 0 \pmod{6}$
(c) $7 \equiv 0 \pmod{7}$ (d) $7 \equiv 0 \pmod{8}$

9. Which one is the smallest pseudoprime to base 3

- (a) 91 (b) 217
(c) 341 (d) 561

10. Fermat's theorem says _____

- (a) $a^{p-1} \equiv 0 \pmod{p}$
(b) $a^{p-1} \equiv 1 \pmod{p}$
(c) $a^{p-1} \equiv -1 \pmod{p}$
(d) $a^p \equiv p-1 \pmod{p}$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) State and prove Archimedean property.

Or

(b) Prove that $\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$, $1 \leq k \leq n$.

12. (a) If $a|c$ and $b|c$ with gcd $(a, b) = 1$, then prove that $ab|c$.

Or

(b) Prove that any positive integers a and b ,
gcd (a, b) lcm $(a, b) = ab$.

13. (a) Prove that the number $\sqrt{2}$ is irrational.

Or

(b) Prove that there are an infinite number of
primes of the form $4n+3$.

14. (a) Prove that 41 divides $2^{20} - 1$.

Or

(b) Solve $9x \equiv 21 \pmod{30}$.

15. (a) State and prove Fermat's theorem.

Or

(b) Using Fermat's method to factorize the
number 119143.

Page 3 Code No. : 20068 E

Page 4 Code No. : 20068 E
[P.T.O.]

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(2n+1)(n+1)}{6}$$

using induction.

Or

- (b) State and prove binomial theorem.

17. (a) State and prove division algorithm.

Or

- (b) Solve the Diophantine equation
 $24x + 138y = 18$.

18. (a) State and prove fundamental theorem of Arithmetic.

Or

- (b) If p_n is the n^{th} prime, then prove that
 $p_n \leq 2^{2^{n-1}}$.

19. (a) State and prove Chinese remainder theorem.

Or

- (b) Explain the Basic properties of congruence.

20. (a) State and prove Wilson's theorem.

Or

- (b) Prove that if n is an odd pseudoprime, then
 $M_n = 2^n - 1$ is a larger one.
-